Inflation and the Fine-Tuning Problem

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We describe a recently derived stochastic approach to inflaton dynamics which can address some serious problems associated with conventional inflationary theory. Using this theory we derive an exact solution to the stochastic dynamics for the case of a $\lambda\phi^4$ potential and use it to study the primordial density fluctuations. We find that on both sub and super-horizon scales the theory predicts gaussian fluctuations to a very high accuracy along with a near scale-invariant spectrum. Of most interest is that the amplitude constraint is found to be satisfied for $\lambda \sim 10^{-5}$ rather than $\lambda \sim 10^{-14}$ of the conventional theory. This represents a dramatic easing of the fine-tuning constraints, a feature likely to generalize to a wide range of potentials.

The inflationary universe scenario asserts that, at some very early time, the universe went through a de Sitter phase expansion with scale factor $a(t)$ growing as e^{Ht}. Inflation is needed because it solves the horizon, flatness and monopole problems of the very early universe and also provides a mechanism for the creation of primordial density fluctuations. For these reasons it is an integral part of the standard cosmological model [1].

The inflationary phase is driven by a quantum scalar field with a potential $V(\Phi)$, that can take on many different forms that satisfy the 'slow roll' conditions. In the conventional approach to inflaton dynamics [1], the inflaton field Φ is first split into a spatially homogeneous piece and an inhomogeneous piece

$$\Phi(\mathbf{x}, s) = \phi(s) + \psi(\mathbf{x}, s). \tag{1}$$

The dynamics of the ϕ is then postulated to obey the classical 'slow roll' equation of motion

$$\dot{\phi} + \frac{V'(\phi)}{3H} = 0. \tag{2}$$

This equation governs the dynamics of ϕ which drives the inflationary phase. It is also possible to discuss the generation of primordial density fluctuations using ψ. Assuming that $\phi \gg \psi$, it can be shown that ψ is described by a free massless minimally coupled quantum scalar field. During exponential inflation the quantum fluctuations of ψ grow as [2]

$$\langle \psi^2 \rangle = (2\pi)^{-2} H^3 t. \tag{3}$$

These quantum fluctuations are then identified with the classical fluctuations which generate primordial density fluctuations [1,3]. It is important to note here that interactions between the coarse-grained field ϕ and its fluctuations ψ are ignored. This is possible in this approach because the density fluctuations are directly identified with $\langle \psi^2 \rangle$.

Consistent with the conventional approach above is the 'Stochastic Inflation' program initiated by Starobinsky [4] and further developed by others [5]. In this case the field ϕ obeys

$$\dot{\phi} + \frac{V'(\phi)}{3H} = \frac{H^{3/2}}{2\pi} F_w(t), \tag{4}$$

where $F_w(t)$ is a zero mean gaussian white noise source of unit amplitude. In this case ϕ describes the field Φ coarse-grained over a volume determined by the de Sitter Hubble radius. We will refer to ϕ as the local order parameter. In this method the observable universe is comprised of many patches each with its own local order parameter whose dynamics obeys (4). Spatial inhomogeneities arise because the local order parameter in each patch can take on different values by virtue of the noise in (4). Equation (4) has been the basis for many applications including studies of the generation of primordial density fluctuations [6,7] and the very large scale structure of the universe [8].

A problem with the conventional approach is that it is assumed, without justification, that the local order parameter ϕ can be treated as a classical order parameter, and that the quantum fluctuations of ψ are equivalent to classical fluctuations. Since ϕ and ψ are treated as decoupled closed quantum systems it is impossible for this method to explain the quantum-to-classical transition of ϕ and ψ from first principles. Another more serious problem comes from directly identifying the quantum fluctuations $\langle \psi^2 \rangle$, with the classical fluctuations that generate primordial density fluctuations. This scheme leads to an overproduction of primordial density fluctuations which can only be avoided by unnaturally fine-tuning the coupling constants in the inflaton potential. This is the well known fine-tuning problem of inflation.

Several authors have previously suggested that these problems arise because the conventional approach to calculating primordial density fluctuations is inconsistent with the established methods of non-equilibrium statistical physics. This was first pointed out by Hu and Zhang

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[9] and further developed in [10] (see also Lombardo and Mazzitelli [11] and Morikawa [12]). Morikawa [13] first suggested that this inconsistency was the origin of the fine-tuning problem. Calzetta and Hu [14] and more recently Calzetta and Gonorazky [15] independently and in much greater detail addressed this issue for a $\lambda\phi^4$ theory.

While the conventional approach may be the only possible one for a free field, in [16] an alternative has been developed for interacting fields which does address the problems outlined above. The theory is similar in style to the conventional stochastic inflation program but differs in a fundamental way. In this theory we no longer identify the quantum fluctuations $\langle\psi^2\rangle$ directly with the classical fluctuations that generate primordial density fluctuations. The new role of the field ψ is to provide a noise source (via backreaction) in the quantum dynamics of the local order parameter ϕ. This is nothing but an application of the well known quantum Brownian motion paradigm of non-equilibrium statistical physics (see [17] and references therein). The field ψ plays the role of an environment which couples to the system ϕ and indirectly generates fluctuations $\delta\phi$ in the system. This environmental noise will generate quantum decoherence which is the process that leads to entropy generation and the quantum-to-classical transition of the order parameter and its fluctuations. We then identify the resulting classical fluctuations of the local order parameter $\delta\phi$, as those which lead to density fluctuations, rather than the quantum fluctuations derived directly from $\langle\psi^2\rangle$. Clearly in this approach the interaction between ϕ and ψ is critical. As well as addressing the quantum-to-classical transition problem, this theory leads to a dramatic easing of the fine tuning constraints, a problem that has plagued the conventional approach to inflaton dynamics.

In the classical limit of this theory, the dynamics of the local order parameter is described by [16]

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \frac{H^2}{8\pi^3}V'''(\phi)F_c(t) \tag{5}$$

where F_c is a colored gaussian noise of unit amplitude with a correlation time of the order H^{-1}. The origin of the noise is the backreaction of quantum fluctuations with wavelengths shorter than the coarse-graining scale. The noise correlation function is ultraviolet *finite* and also turns out to be independent of any ultraviolet cutoff. The noise is of a multiplicative nature because its origin is the mode-mode coupling induced by the self-interaction of the inflaton. For a free field the stochastic term vanishes. This is because the environment ψ and the system ϕ now decouple and the conventional situation is recovered. Significant simplification of (5) was obtained by invoking the standard slow roll assumptions. This made it possible to show that neglecting the potential renormalization and non-local dissipation terms was a good first order approximation in the early slow roll phase.

The coarse-graining scale must be greater than the Hubble radius. This condition allows us to ignore the spatial gradient term in the system sector. The theory is essentially independent of the coarse-graining scale when this condition is met. This robustness to the nature of the coarse-graining is an important virtue of the theory. We also ignore information about spatial correlations between the order parameters of different regions. This allows a description based on a single degree of freedom. Using the slow roll assumptions we can drop the inertial term in (5) and approximate the colored noise by a white noise. In this case we find that (5) becomes

$$\dot{\phi} + \frac{V'(\phi)}{3H} = \frac{H^{1/2}}{\sqrt{864}\pi^3}V'''(\phi)F_w(t). \tag{6}$$

F_w is a white noise of unit amplitude that is interpreted in the Stratonovich sense (since it is an approximation to a colored noise). We also interpret the noise in (4) the same way, though in this case one is also free to use the Ito interpretation. Equation (6) is the result we will use in this letter.

Equation (6) is valid only as long as the slow-roll approximation is valid. This approximation can be shown to be valid if the slow-roll parameters ϵ and $|\eta|$ are both less than 1, where [1]

$$\epsilon(\phi) = \frac{m_{pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \tag{7}$$

and

$$\eta(\phi) = \frac{m_{pl}^2}{8\pi}\left[\frac{V''(\phi)}{V(\phi)} - \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2\right]. \tag{8}$$

Consider the scalar field at the initial value ϕ_0. The field evolves according to (2) to the minimum of the potential, where it then oscillates and decays into other particles (reheating). Inflation ends and reheating commences at a field value ϕ_e where the first slow-roll parameter $\epsilon(\phi_e)$ is unity:

$$\epsilon(\phi_e) = \frac{m_{pl}^2}{16\pi}\left(\frac{V'(\phi_e)}{V(\phi_e)}\right)^2 = 1, \tag{9}$$

with $\epsilon(\phi) < 1$ during inflation. Our discussion in this letter will be restricited to the potential

$$V(\phi) = \lambda\phi^4/4 \tag{10}$$

for which equation (6) will be exactly solvable. For the potential (10) we find from (9) that $\phi_e = 0.56m_{pl}$. This sets a lower bound on ϕ. The second slow roll condition is always satisfied up to this value.

The number of e-folds of inflation which occur when the field evolves from ϕ to ϕ_e is [1]

$$N(\phi) = \frac{8\pi}{m_{pl}^2}\int_{\phi_e}^{\phi}\frac{V(\phi')}{V'(\phi')}d\phi'. \tag{11}$$

Smoothness on scales comparable to the current observable universe requires $N \geq 60$. This places a lower limit on the initial field value $\phi_0 \geq \phi_{60}$, where $N(\phi_{60}) = 60$. For the potential (10) we find from (11) that

$$N(\phi) = \frac{\pi}{m_{pl}^2}(\phi^2 - \phi_e^2). \tag{12}$$

From this we can deduce $\phi_{60} = 4.4 m_{pl}$. In this letter, as is common in most inflation models, we assume that the observable universe leaves the horizon during inflation at 60 Hubble times before the end of inflation, i.e when the inflaton field has the value ϕ_{60}. The smallest scale for which density fluctuations can be probed ($\sim 1Mpc$) will leave the horizon about 10 Hubble times after the observable universe. From (12) we find this corresponds to the field value $\phi_{50} = 4.0 m_{pl}$. Significant is that the critical field values ϕ_e and ϕ_{60-50} are independent of λ. The initial field ϕ_0 cannot be arbitrarily large. It must also satisfy $\phi_0 \leq \phi_{pl}$ where ϕ_{pl} is the field at the Planck boundary which satisfies $V(\phi_{pl}) = m_{pl}^4$. While ϕ_e and ϕ_{50-60} are independent of λ, this is clearly not so for ϕ_{pl}.

In this letter we wish to calculate the statistical properties of observable density fluctuations predicted by (6) for the potential (10). Our aim is to demonstrate that this new approach to inflaton dynamics is consistent with the observed near gaussian and scale invariant density fluctuations. It is well known from gauge-invariant analysis that the amplitude of a density fluctuation that crosses back inside the horizon after inflation, can be deduced from the quantity [1,3]

$$\frac{\delta\rho}{\rho} = \delta\phi \frac{H(\phi)}{\dot{\phi}}, \tag{13}$$

evaluated at the time the fluctuation scale of interest crossed outside the horizon during inflation. The power spectrum $\Delta(k)$ is related to the mean square density fluctuations $\delta\rho/\rho$ via

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \int_{-\infty}^{\infty} \Delta(k) d\ln k \tag{14}$$

from which we obtain

$$\Delta(k) = \frac{d}{d\ln k}\left(\frac{\delta\rho}{\rho}\right)^2. \tag{15}$$

The power spectrum is just the contribution, at a given time, to the mean square fluctuations generated in a Hubble time. We are interested in the power spectrum over observable scales. Therefore we will evaluate the RHS of (15) at ϕ_{60} which is the classical field value at the time the scale of the observable universe leaves the horizon during inflation. It is usual to assume that the density fluctuation power spectrum $\Delta(k)$ can, within the observable range of k, be written as

$$\Delta(k) = Ak^{n-1} \tag{16}$$

where A and n are the amplitude and spectral index of the density fluctuations. For $n = 1$ we have a scale-invariant power spectrum of density fluctuations. From (16) we find that the spectral index can be determined from the power spectrum by

$$n = 1 + \frac{1}{\Delta(k)}\frac{d\Delta(k)}{d\ln k}. \tag{17}$$

Clearly our first task is to calculate (13) for which we need to solve (6) in order to obtain $\delta\phi$. As is commonly done in studies of the Starobinsky equation (4) [6,7], we include the effects of backreaction by simply assuming that H is slowly varying as

$$H^2(\phi) = \frac{8\pi}{3m_{pl}^2}V(\phi). \tag{18}$$

This is possible since H changes little over a Hubble time due to the slow rolling of the inflaton. Upon substituting (10) and (18) into (6), and changing to the dimensionless variables $x = \phi/m_{pl}$ and $\tau = t m_{pl}$, we obtain

$$dx = -fx d\tau + gx^2 F_w(\tau) d\tau \tag{19}$$

where

$$f = \sqrt{\frac{\lambda}{6\pi}}, \quad g = \frac{1}{\sqrt{24\pi^3}}\left(\frac{2\pi}{3}\right)^{1/4}\lambda^{5/4}. \tag{20}$$

With the new variable $\chi = -1/x$ we find that (19) becomes

$$d\chi = f\chi + gF_w(\tau). \tag{21}$$

This equation now describes an Ornstein-Uhlenbeck process for which the solution is

$$\chi(\tau) = \chi_0 e^{f\tau} + g e^{f\tau}\int_0^{\tau} e^{-fs} F_w(s) ds. \tag{22}$$

Therefore our solution to (19) is

$$x(\tau) = \frac{x_c(\tau)}{1 - gx_0\int_0^{\tau} e^{-fs}F_w(s)ds}, \tag{23}$$

where the classical deterministic solution $x_c(\tau)$ is

$$x_c(\tau) = x_0 e^{-f\tau} \tag{24}$$

with x_0 as the initial field value. To obtain the fluctuations $\delta\phi$ we make a gaussian approximation

$$x(\tau) \simeq x_c(\tau)\left(1 + gx_0\int_0^{\tau} e^{-fs}F_w(s)ds\right) \tag{25}$$

to (23) which will be justified later. From this we obtain

$$(\delta x)^2 = \langle (x(\tau) - x_c(\tau))^2 \rangle$$
$$\simeq \frac{\lambda^2}{24\pi^5} x_c^2(\tau)\big(x_0^2 - x_c^2(\tau)\big). \tag{26}$$

We can compare this result directly to that predicted by the Starobinsky equation (4). Upon substituting (10) and (18) into (4), we find the exact solution to the Starobinsky equation is

$$x(\tau) = \frac{x_c(\tau)}{\sqrt{1 - 2hx_0^2 \int_0^\tau e^{-2fs} F_w(s)ds}}$$
$$\simeq \frac{x_c(\tau)}{1 - hx_0^2 \int_0^\tau e^{-2fs} F_w(s)ds}, \tag{27}$$

where

$$h = \frac{1}{2\pi}\left(\frac{2\pi\lambda}{3}\right)^{3/4}. \tag{28}$$

Making the same gaussian approximation as previously we find that

$$(\delta x_s)^2 \simeq \frac{\lambda}{12} x_c^2(\tau)\big(x_0^4 - x_c^4(\tau)\big) \tag{29}$$

where we use the s subscript to denote quantities evaluated from the solution to the Starobinsky equation.

Using (26),(18) and (24) we find that (13) becomes

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \frac{1}{6\pi^3}\lambda^2 x_c^4(\tau)\big(x_0^2 - x_c^2(\tau)\big). \tag{30}$$

To calculate the spectrum (15) we need to relate the classical field value at some time to the scale k that is crossing the horizon at the same time. We do this by first considering the number of e-folds $N(k)$ between the horizon crossing of a scale k and the end of inflation. We are assuming that the scale of the observable universe left the horizon 60 Hubble times before the end of inflation. We can therefore write $N(k)$ as [1]

$$N(k) = 60 + \ln(k_*/k) \tag{31}$$

where k_* is the scale of the current observable universe. From (31) and (12) we find that

$$x_c^2(\tau) = \frac{1}{\pi}\big[60 + \ln(k_*/k)\big] + x_e^2. \tag{32}$$

This determines the value of the field at the time a scale k crosses the horizon. This is the result we will use to calculate the spectrum. Substituting (30) into (15) and using (32) we find

$$\Delta(k) = \frac{\lambda^2}{6\pi^4} x_c^2(\tau)\big(3x_c^2(\tau) - 2x_0^2\big). \tag{33}$$

The explicit k dependence of this function is contained in $x_c(\tau)$ via equation (32). The spectrum has only a very weak logarithmic k dependence. This will clearly give rise to a near scale-invariant spectrum. We can calculate the spectral index from (33) by using (17) and setting x_c and x_0 to x_{60}. Observable constraints on the amplitude of density fluctuations require setting the LHS of (33) to 10^{-10}. Setting x_c and x_0 to x_{60} allows us to calculate a value for λ. The results are

$$\lambda = 1.2 \times 10^{-5}, \quad n = 0.93. \tag{34}$$

We can perform an exactly analogous calculation using the fluctuations (29) derived from the exact solution of the Starobinsky equation. We find the familiar result

$$\lambda_s = 6.6 \times 10^{-15}, \quad n_s = 0.92. \tag{35}$$

We see that equation (6) leads to a dramatic easing of the fine-tuning required, but still gives a near scale-invariant spectrum of density fluctuations.

Also of great interest is a measure of how much the probability distribution of fluctuations deviates from gaussian. When the magnitude of the stochastic term in the denominator of (23) is small ($\ll 1$), it becomes possible to make the gaussian approximation (25) to the exact solution (23). We are therefore led to define

$$d = \frac{gx_0}{\sqrt{2f}} = \frac{x_0\lambda}{\sqrt{24\pi^5}} \tag{36}$$

as a simple measure of the deviation from gaussian. (The denominator $\sqrt{2f}$ of (36) is the typical fluctuation size of the stochastic term in (23) in the stationary limit.) Using $x_0 = 4.4$, and the result (34) for λ, we find $d = 6 \times 10^{-7}$. We can compare (36) to that derived from the exact solution (27) of the Starobinsky equation. In this case we find

$$d_s = \frac{hx_0^2}{2f^{1/2}} = \left(\frac{\lambda_s}{12}\right)^{1/2} x_0^2. \tag{37}$$

Using the result (35) for λ_s, and $x_0 = 4.4$, we find $d_s = 5\times10^{-7}$. We therefore see that over scales of the observable universe both solutions predict nearly identical and negligable deviations from gaussian.

By assuming $x_0 = x_{60}$ we are simply trying to exclude the effects of fluctuations on scales larger than our present horizon. In actual fact inflation would most likely have been in progress for a long time before the observable universe left the Hubble radius during inflation. In this case we can consider x_0 to be anywhere in the range $4.4 \le x_0 \le x_{pl}$. For the usual self-coupling constant in (35) we find that the Planck boundary is $x_{pl} \simeq 5000$. Because the deviations from gaussian in (37) are proportional to x_0^2 there is a possibility of significant deviations from gaussian fluctuations on super-horizon scales. For the new coupling constant in (34) we have $x_{pl} \simeq 24$. In this case the deviations from gaussian in (36) depend only linearly on x_0. This combined with the much smaller

Planck boundary means that the solution (23) predicts gaussian fluctuations to a high accuracy over both super and sub-horizon scales. We have ignored the effects of boundary conditions that should be imposed at ϕ_e and ϕ_{pl}. These effects will be small and and are unlikely to change these conclusions.

Yi, Vishniac and Minehige [7] have analysed in detail solutions of the Starobinsky equation (in both Ito and Statonovich interpretations) for the model discussed here. They discussed deviations from gaussian in terms of a simple measure for skewness derived from the probability distribution of the exact solution (27). We have also applied this measure to the exact solution (23) and found it to give results consistent with the simpler measure discussed above.

In this letter we showed that the new theory of inflaton dynamics developed in [16] can address the fine-tuning problem yet still predict density fluctuations in general agreement with observations. This was demonstrated for the quartic potential but is likely to generalize to a wide range of potentials. These results were based on the classical limit of the theory. The other great advantage of this theory is that it leads naturally to a description of the inflaton as a quantum open system. This allows the quantum-to-classical transition to occur as a non-equilibrium quantum statistical process (decoherence), rather than being simply postulated as in the conventional approach. Work on this issue is currently in progress.

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