Cosmological Time in Quantum Supergravity

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Abstract

The version of supergravity formulated by Ogievetsky and Sokatchev is almost identical to the conventional $N = 1$ theory, except that the cosmological constant Λ appears as a dynamical variable which is constant only by virtue of the field equations. We consider the canonical quantisation of this theory, and show that the wave function evolves with respect to a dynamical variable which can be interpreted as a cosmological time parameter. The square of the modulus of the wave function obeys a set of simple conservation equations and can be interpreted as a probability density functional. The usual problems associated with time in quantum gravity are avoided.

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One of the most fundamental questions in quantum cosmology is that of identifying a suitable time parameter, with respect to which the dynamics of the Universe can be measured. The conventional Wheeler-DeWitt formulation gives a time-independent quantum theory, and does not suggest any obvious reason why observers should experience the passage of time. Moreover, the absence of any special parametrisation leads to ambiguities when defining transition amplitudes between specified 3-geometries [1]. A related problem is that of choosing an inner product on the Hilbert space of physical states, in the absence of any parameter with respect to which this inner product should be conserved.

Supersymmetry transformations are more fundamental than time translations, in the sense that the latter may be generated by anticommutators of supersymmetry generators. For this reason, it is natural to look to supersymmetry for a solution to the problems outlined above. One is therefore led to consider theories such as $N = 1$ supergravity.

Perhaps the most elegant and economical description of supergravity is that of Ogievetsky and Sokatchev [2]. In this formulation, the supergravity multiplet is obtained from a complex vector superfield by imposing simple gauge conditions together with a type of unimodularity condition on the supercoordinate transformations. The resulting theory is identical in most respects to conventional supergravity (as described by Wess and Bagger, for example [3]).

However there is a subtle difference between the multiplet of Ogievetsky and Sokatchev and that of the conventional theory, which does not appear to have been previously exploited [4]. The conventional supergravity multiplet contains a scalar density M which is treated as an independent auxiliary field; however, in the theory of Ogievetsky and Sokatchev M is given in terms of a vector density whose longitudinal component is actually dynamical. It turns out that this additional dynamical degree of freedom is essentially the cosmological "constant" Λ, which in this case is constant only by virtue of the equations of motion.

A dynamical Λ also appears in general relativity if a unimodular condition is imposed on the metric before the variation of the fields [5, 6, 7]. In that case, Λ is found to be canonically conjugate to a parameter which is naturally interpreted as cosmological time.

It is shown below that a similar result is obtained in the Ogievetsky and Sokatchev version of supergravity. However in this case, unlike that of unimodular general relativity, the result is achieved without the imposition of any ad-hoc constraints on the physical fields. Moreover the cosmological time is a dynamical variable determined by the fields on a given hypersurface, while in unimodular general relativity it depends on the entire past history of the fields prior to this hypersurface.

The Lagrangian for $N = 1$ supergravity is

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}e\mathcal{R} + \frac{1}{2}e\epsilon^{klmn}e_\ell{}^a(\bar{\psi}_k\bar{\sigma}_a\widetilde{\mathcal{D}}_m\psi_n - \psi_k\sigma_a\widetilde{\mathcal{D}}_m\bar{\psi}_n) + \frac{1}{3}eb^a b_a \\ &\quad -e\lambda^*(M + \psi_a\sigma^{ab}\psi_b) - e\lambda(M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b) - \frac{1}{3}eM^*M \end{aligned} \tag{1}$$

where λ, λ^* are externally specfied constants, e is the determinant of the tetrad $e_{\hat{\ell}}{}^a$, and $\mathcal{R}$ is the Ricci scalar [1]. Conventionally, M and M^* are treated as auxiliary fields and eliminated using their equations of motion $M = -3\lambda$ and $M^* = -3\lambda^*$. This leads to a theory with fixed gravitino mass $m = |\lambda|$ and cosmological constant $\Lambda = -3m^2$.

In the formulation of Ogievetsky and Sokatchev, the cosmological constant arises in quite a different way [2]. In this approach M is not an independent field, but is given by the expression

$$M = e^{-1}\partial_m M^m - \psi_a\sigma^{ab}\psi_b \tag{2}$$

where the complex vector density M^m now plays the part of the auxiliary field. The λ,λ^* terms appearing in the Lagrangian (1) are then just total divergences and can be dropped. Varying M^m (rather than M as in the usual approach) leads to the field equations $\partial_m M = \partial_m M^* = 0$. The equations of motion for the remaining fields

[1] We use the notation and conventions of [3], with spacetime coordinates denoted by letters from the middle of the alphabet. Note that $\mathcal{R}$ differs by a sign from the curvature scalar defined in [8] and [9].

are identical to those of conventional supergravity with gravitino mass $m = \frac{1}{3}|M|$ and cosmological constant $\Lambda = -3m^2$. In the present formulation, however, M is not an externally specified constant but a dynamical variable which is constant only on-shell. This difference is vital, because it means that the quantum theory admits linear superpositions of states with different values of the gravitino mass and cosmological constant.

The Ogievetsky-Sokatchev model also has two other important new features. Firstly, it is invariant under the gauge transformation

$$M^m \mapsto M^m + \delta M^m, \qquad \delta M^m = e\epsilon^{mjkl}\partial_j A_{kl} \tag{3}$$

where A_{kl} is an anti-symmetric tensor density. Secondly, owing to the elimination of the λ, λ^* terms from the action, the Ogievetsky-Sokatchev model also has the global $U(1)$ symmetry

$$\psi \mapsto e^{i\phi}\psi, \qquad M^m \mapsto e^{2i\phi}M^m. \tag{4}$$

In the conventional formulation of supergravity this symmetry is broken for $\lambda \neq 0$.

We wish to determine whether the dynamical nature of Λ in Ogievetsky-Sokatchev supergravity leads to a natural definition of cosmological time, as in unimodular general relativity. To this end we now consider the canonical formulation of the theory, focussing on those points which arise from the replacement of the scalars M, M^* by the vector densities M^m, M^{*m} as independent fields. (See [9] for a canonical description of the conventional theory.)

We begin by making a 3+1 space-time split, with spacelike coordinates denoted by hatted latin indices $(\hat{m}, \hat{n}, \ldots)$ and the timelike coordinate denoted by the index t. For example, the tetrad $e^n{}_a$ is split into a timelike part $e^t{}_a = -n_a/N$ and a spatial part $e^{\hat{n}}{}_a = n_a(N^{\hat{n}}/N) + h^{\hat{n}\hat{m}}e_{\hat{m}a}$ where N is the lapse funtion, $N^{\hat{m}}$ is the shift vector, $h_{\hat{m}\hat{n}} = e_{\hat{m}}{}^a e_{\hat{n}a}$ is the spatial 3-metric, $h^{\hat{m}\hat{n}}$ is its inverse, and n_a is defined so that $n_a n^a = -1$ and $e_{\hat{m}}{}^a n_a = 0$.

Similarly, the vector densities M^m, M^{*m} consist of spatial components $M^{\hat{m}}$, $M^{*\hat{m}}$ which are non-dynamical, and time-like components M^t, M^{*t} which *are* dynamical and have canonical momenta $p = -\frac{1}{3}M^*$, $p^* = -\frac{1}{3}M$.

Within any spatial hypersurface $\Sigma(t)$, the spatially varying part of M^t can be gauged away by a transformation of the form (3). (One simply chooses the gauge transformation parameter as $A_{\hat{k}\hat{l}} = e^{-1}\epsilon_{t\hat{k}\hat{l}\hat{m}}B^{\hat{m}}$ where $2\partial_{\hat{n}}\partial_{\hat{m}}B^{\hat{m}} + \partial_{\hat{n}}M^t = 0$.) After the removal of the gauge degrees of freedom, all that remains of M^t is the spatially constant part whose integral is

$$q(t) \equiv \int_{\Sigma(t)} M^t. \tag{5}$$

The Rarita-Schwinger fields are also split into timelike parts ψ_t, $\bar{\psi}_t$ and spatial parts $\psi_{\hat{m}}$, $\bar{\psi}_{\hat{m}}$. It is convenient to eliminate ${\psi_t}^\alpha$, $\bar{\psi}_{t\dot{\alpha}}$ in favour of the spinor

$$\chi^\alpha \equiv {\psi_t}^\alpha - \frac{2}{3}Nh^{\hat{m}\hat{n}}e_{\hat{n}a}n_b{\psi_{\hat{m}}}^\beta {\sigma^{ab}}_\beta{}^\alpha - \frac{1}{3}N^{\hat{m}}{\psi_{\hat{m}}}^\alpha - \frac{1}{3}N^{\hat{m}}e_{\hat{m}c}h^{\hat{\ell}\hat{n}}e_{\hat{n}a}{\psi_{\hat{\ell}}}^\beta {\sigma^{ac}}_\beta{}^\alpha \tag{6}$$

and its hermitian conjugate $\bar{\chi}_{\dot{\alpha}}$. This definition leads to the useful identity $e\psi_a\sigma^{ab}\psi_b = -2h^{\frac{1}{2}}(\chi\sigma^{ab}\psi_{\hat{m}})n_a h^{\hat{m}\hat{n}}e_{\hat{n}b}$, where $h \equiv \det[h_{\hat{m}\hat{n}}]$.

We also eliminate N in favour of the variable $\widetilde{N} \equiv Nh^{\frac{1}{2}}$. The Hamiltonian is then

$$\begin{aligned} H \;=\; \int d^3x\Big\{ & M^{\hat{m}}\partial_{\hat{m}}p + M^{*\hat{m}}\partial_{\hat{m}}p^* + {\omega_t}^{ab}J_{ab} \\ & +\widetilde{N}[h^{-\frac{1}{2}}\mathcal{H} - 3p^*p + \frac{2}{3}h^{-\frac{1}{2}}h^{\hat{m}\hat{n}}e_{\hat{n}a}n_b({\psi_{\hat{m}}}^\beta {\sigma^{ab}}_\beta{}^\alpha S_\alpha + \bar{\psi}_{\hat{m}\dot{\beta}}\bar{\sigma}^{ab\dot{\beta}}{}_{\dot{\alpha}}S^{\dot{\alpha}})] \\ & +N^{\hat{m}}[\mathcal{H}_{\hat{m}} + \frac{1}{3}({\psi_{\hat{m}}}^\alpha + e_{\hat{m}c}h^{\hat{\ell}\hat{n}}e_{\hat{n}a}{\psi_{\hat{\ell}}}^\beta {\sigma^{ac}}_\beta{}^\alpha)S_\alpha + \frac{1}{3}(\bar{\psi}_{\hat{m}\dot{\alpha}} + e_{\hat{m}c}h^{\hat{\ell}\hat{n}}e_{\hat{n}a}\bar{\psi}_{\hat{\ell}\dot{\beta}}\bar{\sigma}^{ac\dot{\beta}}{}_{\dot{\alpha}})\bar{S}^{\dot{\alpha}}] \\ & +\chi^\alpha(S_\alpha - 2ph^{\frac{1}{2}}n_a h^{\hat{m}\hat{n}}e_{\hat{n}b}{\sigma^{ab}}_\alpha{}^\beta\psi_{\hat{m}\beta}) \\ & +\bar{\chi}_{\dot{\alpha}}(\bar{S}^{\dot{\alpha}} - 2p^*h^{\frac{1}{2}}n_a h^{\hat{m}\hat{n}}e_{\hat{n}b}\bar{\sigma}^{ab\dot{\alpha}}{}_{\dot{\beta}}{\bar{\psi}_{\hat{m}}}{}^{\dot{\beta}})\Big\} \end{aligned} \tag{7}$$

where the expressions for the quantities $\mathcal{H}(x)$, $\mathcal{H}_{\hat{m}}(x)$, $S_\alpha(x)$, $\bar{S}^{\dot{\alpha}}(x)$, and $J_{ab}(x)$ are familiar from the standard formulation of canonical supergravity [9], in which they play the roles of Hamiltonian, momentum, supersymmetry and Lorentz constraints respectively.

The momenta conjugate to $\widetilde{N}(x)$, $N^{\hat{m}}(x)$, $\chi^\alpha(x)$, $\bar{\chi}_{\dot{\alpha}}(x)$, ${\omega_t}^{ab}(x)$, $M^{\hat{m}}(x)$ and $M^{*\hat{m}}(x)$ are found to vanish, indicating that these variables act as Lagrange multipliers in (7). The requirement that H be stationary with respect to these Lagrange multipliers then leads to a set of secondary constraints at each point x. In particular, $M^{\hat{m}}$ and $M^{*\hat{m}}$ enforce the constraints $\partial_{\hat{m}} p = \partial_{\hat{m}} p^* = 0$ indicating that p, p^* are spatially constant. In fact the parameters $p(t), p^*(t)$ are simply the momenta conjugate to the canonical coordinates $q(t), q^*(t)$.

The quantity $\Lambda = -3p^*p$ has physical significance, as it plays the role of the cosmological constant in the field equations derived from (1). We therefore eliminate $p(t), p^*(t)$ in favour of real dynamical variables $\Lambda(t)$ and $\theta(t)$, defined so that $p = (-\Lambda/3)^{\frac{1}{2}} e^{i\theta}$. The equations of motion then ensure that Λ and θ remain constant along classical trajectories.

In unimodular general relativity, Λ is canonically conjugate to a variable which can be interpreted as the cosmological time parameter. In the present case, the variables $\Lambda(t)$ and $\theta(t)$ can be identified respectively as the momenta Π_T, Π_Q conjugate to the new canonical coordinates

$$
\begin{aligned}
T(t) &= -\frac{1}{6p^*p}(pq + p^*q^*) && (8)\\
Q(t) &= i(pq - p^*q^*) && (9)
\end{aligned}
$$

as can be seen by calculating the Dirac bracket relations.

The analogy with unimodular general relativity raises the hope that the new variable T might play the role a cosmological time parameter. This hope is fulfilled. The equations of motion derived from (1) imply that

$$
\frac{dT}{dt} = \int_{\Sigma(t)} d^3x \left[e - \frac{h^{\frac{1}{2}}}{\Lambda}(p\chi\sigma^{ab}\psi_{\hat{m}} + p^*\bar{\chi}\bar{\sigma}^{ab}\bar{\psi}_{\hat{m}})n_a h^{\hat{m}\hat{n}} e_{\hat{n}b} \right]. \qquad (10)
$$

Integrating and making use of the supersymmetry constaints (enforced in (7) by the Lagrange multipliers $\chi^\alpha, \bar{\chi}_{\dot{\alpha}}$) one obtains the weak equalities [10]

$$
T(t) \approx \int_{\mathcal{M}} d^4x \left[e - \frac{1}{2\Lambda}(\chi^\alpha S_\alpha + \bar{\chi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}}) \right] \qquad (11)
$$

where $\mathcal{M}$ denotes the 4-volume between the hypersurfaces $\Sigma(t_0)$ and $\Sigma(t)$, with t_0 defined so that $T(t_0) = 0$. Similarly, one finds that

$$
Q(t) - Q(t_0) \approx -i \int_{\mathcal{M}} d^4x\, (\chi^\alpha S_\alpha - \bar{\chi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}}). \qquad (12)
$$

In order to extract the gauge-invariant content of $T(t)$ and $Q(t)$, we may now choose a gauge with $\chi^\alpha = \bar{\chi}_{\dot{\alpha}} = 0$ everywhere. The equations of motion then imply that $Q(t)$ is constant while $T(t)$ is the invariant 4-volume of spacetime preceeding the hypersurface $\Sigma(t)$. In this gauge, therefore, $T(t)$ coincides classically with the cosmological time parameter that arises both in unimodular general relativity [5, 6, 7] and in Sorkin's sum-over-histories approach [11]. Note that T is a monotonically increasing function along any classical trajectory and so can indeed be used to parametrise this trajectory.

As well as imposing gauge conditions on $\chi^\alpha\ \bar{\chi}_{\dot{\alpha}}$, it is also necessary to fix the other Lagrange multipliers $\widetilde{N}(x)$, $N^{\hat{m}}(x)$ and $\omega_t{}^{ab}(x)$ at each spacetime point x in order that the equations of motion have a unique solution and the classical evolution is well-defined. (It is immaterial *how* these Lagrange multipliers are chosen, provided that $\widetilde{N} > 0$.) Then through each point in phase space at which the constraints hold, there passes a unique classical trajectory parametrised by T. The closure of the Dirac constraint algebra ensures that the whole trajectory will be confined to the region in phase space where the classical constraints are satisfied.

Having removed the ambiguities from the classical evolution equations by specifying the Lagrange multipliers, we now proceed to the quantum theory. The constraints become conditions on the wave function Ψ, their precise form depending on the representation used. An obvious choice is the (Λ, θ) representation, with a wave function $\Psi(\Lambda, \theta; e_{\hat{m}}{}^a, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}]$. However, more insight into the nature of the supersymmetry constraints is obtained by switching to the (Λ, Q) representation, with $\Psi(\Lambda, \theta; e_{\hat{m}}{}^a, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}]$ decomposed as a linear combination of eigenmodes Ψ_N of the operator $Q = i\hbar\partial/\partial\theta$:

$$\Psi(\Lambda, \theta; e_{\hat{m}}{}^a, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}] = \sum_{N=-\infty}^{\infty} e^{-iN\theta}\, \Psi_N(\Lambda; e_{\hat{m}}{}^a, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}]. \tag{13}$$

At each point x there is a family of constraints, which in this representation have the form

$$h^{-\frac{1}{2}}\mathcal{H}\Psi_N \quad = \quad -\left[\frac{2}{3}h^{-\frac{1}{2}}h^{\hat{m}\hat{n}}e_{\hat{n}a}n_b(\psi_{\hat{m}}{}^\beta\sigma^{ab}{}_\beta{}^\alpha S_\alpha + \bar{\psi}_{\hat{m}\dot{\beta}}\bar{\sigma}^{ab\dot{\beta}}{}_{\dot{\alpha}}\bar{S}^{\dot{\alpha}}) + \Lambda\right]\Psi_N \tag{14}$$

$$\begin{aligned}
\mathcal{H}_{\hat{m}}\Psi_N &= -\frac{1}{3}\Big[(\psi_{\hat{m}}{}^{\alpha} + e_{\hat{m}c}h^{\hat{\ell}\hat{n}}e_{\hat{n}a}\psi_{\hat{\ell}}{}^{\beta}\sigma^{ac}{}_{\beta}{}^{\alpha})S_{\alpha} \\
&\qquad + (\bar{\psi}_{\hat{m}\dot{\alpha}} + e_{\hat{m}c}h^{\hat{\ell}\hat{n}}e_{\hat{n}a}\bar{\psi}_{\hat{\ell}\dot{\beta}}\bar{\sigma}^{ac\dot{\beta}}{}_{\dot{\alpha}})\bar{S}^{\dot{\alpha}}\Big]\Psi_N \qquad (15) \\
S_{\alpha}\Psi_N &= 2\left(-\frac{\Lambda}{3}\right)^{\frac{1}{2}} h^{\frac{1}{2}} n_a h^{\hat{m}\hat{n}} e_{\hat{n}b}\sigma^{ab}{}_{\alpha}{}^{\beta}\psi_{\hat{m}\beta}\Psi_{N+1} \qquad (16) \\
\bar{S}^{\dot{\alpha}}\Psi_N &= 2\left(-\frac{\Lambda}{3}\right)^{\frac{1}{2}} h^{\frac{1}{2}} n_a h^{\hat{m}\hat{n}} e_{\hat{n}b}\bar{\sigma}^{ab\dot{\alpha}}{}_{\dot{\beta}}\bar{\psi}_{\hat{m}}{}^{\dot{\beta}}\Psi_{N-1} \qquad (17) \\
J_{ab}\Psi_N &= 0. \qquad (18)
\end{aligned}$$

(For brevity, we have suppressed the spatial dependence of the operators $h^{-\frac{1}{2}}(x)$, $\mathcal{H}(x)$, $\mathcal{H}_{\hat{m}}(x)$, $J_{ab}(x)$, $S_{\alpha}(x)$, $\bar{S}^{\dot{\alpha}}(x)$, $h^{mn}(x)$, $e_{\hat{n}a}(x)$, $n_a(x)$, $\psi_{\hat{m}}{}^{\alpha}(x)$ and $\bar{\psi}_{\hat{m}\dot{\alpha}}(x)$.) The argument Λ is restricted to the non-positive part of the real axis, and each Ψ_N is required to vanish at $\Lambda = 0$. This boundary condition can be derived from the continuity of the wave function in the (p, p^*) representation, and ensures that the operator $T = i\hbar\partial/\partial\Lambda$ is self-adjoint.

An alternative description of the quantum theory can be obtained using the (T, Q) representation, in which the wave function is defined by the Fourier transform

$$\Psi_N(T; e_{\hat{m}}{}^{a}, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}] \equiv \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} d\Lambda\, \exp\left\{\frac{i}{\hbar}T\Lambda\right\}\, \Psi_N(\Lambda; e_{\hat{m}}{}^{a}, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}}] \qquad (19)$$

and Λ is represented by the operator $-i\hbar\partial/\partial T$. In this representation, the Hamiltonian constraints (14) take the form of a family of Schrödinger equations (one at each point x) describing the evolution of the cosmological wave function with respect to the cosmological time parameter T:

$$i\hbar\frac{\partial\Psi_N}{\partial T} = h^{-\frac{1}{2}}\left\{\mathcal{H} + \frac{2}{3}h^{\hat{m}\hat{n}}e_{\hat{n}a}n_b\Big[\psi_{\hat{m}}{}^{\beta}\sigma^{ab}{}_{\beta}{}^{\alpha}S_{\alpha} + \bar{\psi}_{\hat{m}\dot{\beta}}\bar{\sigma}^{ab\dot{\beta}}{}_{\dot{\alpha}}\bar{S}^{\dot{\alpha}}\Big]\right\}\Psi_N. \qquad (20)$$

The momentum and angular momentum constraints have the same form (15,18) in the (T, Q) representation as in the (Λ, Q) representation. However, the supersymmetry constraints are awkward to express in the (T, Q) representation, as they involve the square root of the operator $\Lambda = -i\hbar\partial/\partial T$; when considering these constraints, it is convenient to return to the (Λ, Q) representation.

Assuming that the operator-ordering is chosen so that $\mathcal{H}$ is self-adjoint and $S^{\alpha}, \bar{S}_{\dot{\alpha}}$ are mutually adjoint with respect to the measure on the configuration space

$(e_{\hat{m}}{}^{a}, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}})$, then it follows from the Schrödinger equations (20) that the integral of the quantity $\Psi_N^* \Psi_N$ is conserved with respect to T. Moreover, once we have integrated over the fermionic degrees of freedom and summed over all values of N, this quantity is real and non-negative, and so is naturally interpreted as the probability density function for the Universe.

The momentum constraints (15) imply that Ψ depends only on T and the equivalence class of configurations related to $(e_{\hat{m}}{}^{a}, \bar{\psi}_{\hat{m}}{}^{\dot{\alpha}})$ by spatial diffeomorphisms. An argument by Kuchař [12] can be adapted to show that these equivalence classes are not Dirac observables. However, this is not an obstacle to the interpretation of the wave function suggested above; in a parametrised theory, the wave function arguments need not be observables. (For example, see [13].)

This is illustrated by the parametrised description of particle dynamics in a one-dimensional potential. In this description both $t(\tau)$ and $x(\tau)$ are coordinates, with conjugate momenta $\pi(\tau), p(\tau)$. The Hamiltonian

$$H = N[\pi + \frac{1}{2}p^2 + V(x)]. \tag{21}$$

contains a Lagrange multiplier N enforcing the constraint $\phi \equiv \pi + \frac{1}{2}p^2 + V(x) \approx 0$ which in the quantum theory imposes the condition

$$-\pi\Psi = [\frac{p^2}{2} + V(x)]\Psi. \tag{22}$$

In the (t, x) representation, the momenta are given by the operators $\pi = -i\hbar\partial/\partial t$ and $p = -i\hbar\partial/\partial x$ and so (22) is just the Schrödinger equation. We must therefore adopt the conventional interpretation of $\Psi(t, x)$ as the amplitude at time t for observing the particle at position x.

It is easily seen that the variables t and x have non-vanishing Dirac brackets with the constraint ϕ, and so are not Dirac observables. However this fact does not prevent us from adopting the conventional interpretation of the wave function $\Psi(t, x)$; it simply reflects the breaking of time-translation symmetry by the act of measurement at a definite instant.

Similarly, in the present theory, the non-commutation of the wave function arguments with the Hamiltonian and supersymmetry constraints reflects the fact that measurements are to be made on a definite hypersurface Σ and in a definite supersymmetry gauge. Once again, this does not prevent us from adopting the conventional interpretation of the wave function.

It should be noted here that the dynamical variable T need not be assumed to play any special role in the identification of the hypersurface Σ in the classical theory. (For example, one might specify the embedding of the hypersurface by giving the spacetime coordinates of each point on Σ.) However, since T increases monotically along classical trajectories (at least in the gauge $\chi = \bar{\chi} = 0$), there is a strong temptation to view it as a Heraclitean time parameter labelling the hypersurfaces in some foliation of spacetime [7].

In the context of unimodular gravity, it has been argued that the specification of the cosmological time T is insufficient to identify the hypersurface on which the measurements are to be made [1, 12]. While this is certainly true, the argument no longer holds if the Lagrange multipliers $\widetilde{N}(x)$ and $N^{\hat{m}}(x)$ are fixed in advance, as in the present approach; then the classical evolution is completely determined and each value of T specifies a unique hypersurface. Thus, one can identify a particular hypersurface by specifying a value of the parameter T *and* the Lagrange multipliers at each spacetime point.

It is clear that the choice of Lagrange multipliers makes no difference at all to the quantum constraints (14-18), or to the evolution of the wave function Ψ with respect to the cosmological time parameter T. Hence the the transition amplitude between two specified hypersurfaces is independent of any coordinate conditions which may be imposed on the interpolating spacetimes. The quantum theory therefore escapes the "multiple choice problem", which arises in most other approaches to quantum gravity in which time is found among the canonical variables [1].

In conclusion, we have shown that the canonical quantisation of Ogievetsky-

Sokatchev supergravity leads directly to a time-dependent wave function with a straightforward probabilistic interpretation. It is natural to ask whether this result applies only in $N = 1$ supergravity, or is enjoyed by a wider class of supersymmetric theories. The essential ingredient appears to be the replacement of an auxiliary scalar field by the divergence of a vector field with well-defined supersymmetric transformation properties. This is not a very stringent requirement, and can probably be satisfied by a wide variety of locally supersymmetric theories. Any such theory will have features similar to those outlined above.

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