GLOBAL NILPOTENT CLASS FIELD THEORY

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Let p be an arbitrary, fixed prime, K an algebraic number field and S a set of places of K which ramify in the maximal p-extension of K. This means a $\mathfrak{p} \in S$ is of the form $N(\mathfrak{p}) \equiv 1 \pmod{p}$ or $\mathfrak{p}|p$ or $\mathfrak{p}$ real if $p = 2$.

Let U_S be the subgroup of the group U_K of unit ideles consisting of the ideles with components 1 at the places of S. Let I_K be the idele group of K. Then to the maximal abelian extension of K unramified outside S corresponds in the sense of class field theory the group $\mathfrak{C}_S := I_K/U_S K^\times$. Let $\hat{\mathfrak{C}}_S$ be the pro-p-completion of $\mathfrak{C}_S$. Then the Artin map induces an isomorphism ϕ_1 from $\hat{\mathfrak{C}}_S$ onto the Galois group G_S of the maximal p-extension K_S of K unramified outside S.

The groups G_S are studied in [Ko1970]. We adopt some notation from this book. In particular $\tilde{\mathcal{B}}_S$ denotes the character group of the finite elementary p-group $V_S/K^{\times p}$ with

$$V_S := \{\alpha \in K^\times | (\alpha) = \mathfrak{a}^p, \alpha \in K_\mathfrak{p}^p \text{ for } \mathfrak{p} \in S\},$$

where (α) is the principal ideal associated to α. If S is big enough, then $\tilde{\mathcal{B}}_S = \{0\}$.

In [KoKuLa] we introduced the algebraic setting of a nilpotent clas field theory which in the global case means the following. We associate to $\hat{\mathfrak{C}}_S$ the graded Lie algebra $\bigoplus_{n=1}^{\infty} \mathcal{L}_n(\hat{\mathfrak{C}}_S)$ and to G_S the graded Lie algebra $\bigoplus_{n=1}^{\infty} L_n(G_S) = \bigoplus_{n=1}^{\infty} G_S^{(n)}/G_S^{(n+1)}$, where $\{G_S^{(n)} | n = 1, 2, \dots\}$ is the filtration of G_S by means of the central decending series. The Artin map ϕ_1 induces a map

$$\phi_n : \mathcal{L}_n(\hat{\mathfrak{C}}_S) \to L_n(G_S)$$

again denoted as Artin map.

In particular $\mathcal{L}_2(\hat{\mathfrak{C}}_S)$ is the pro-p-completion of the alternating product $\hat{\mathfrak{C}}_S \wedge \hat{\mathfrak{C}}_s$, which was studied in the local case, i.e. $K^\times \wedge K^\times$, in [Zi1982] and [Zi1983]. In [Zi1982] it was shown as the basis of an arithmetical theory that ϕ_2 is an isomorphism in the local case of class field theory. We want to show here, that this result is true in the global case as well if certain conditions are fulfilled:

Theorem 0.1. *Let $\tilde{\mathcal{B}}_S = \{0\}$ and $\mathfrak{p} \in S$ for one place $\mathfrak{p}$ of K with $\mathfrak{p}|p$. Then the Artin map ϕ_2 is an isomorphism of $\mathcal{L}_2(\hat{\mathfrak{C}}_S)$ onto $L_2(G_S)$.*

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1. The proof of Theorem 0.1 uses the results of [Ko1970], §11, which we remember first.

We introduce some further notations. For any field define $\delta(L) := 1$ if the p-th roots of unity are in L and $\delta(L) := 0$ if not. A place $\mathfrak{p}$ of K is ramified in the maximal p-extension $\overline{K}$ of K only if $N(\mathfrak{p}) \equiv 1 \pmod p$ or $\mathfrak{p}|p$ or $\mathfrak{p}$ real (in the case $p = 2$). Hence in the following we assume that a $\mathfrak{p}$ in S is of one of these types. For technical reasons we introduce the number $e_\mathfrak{p}$ for $\mathfrak{p} \in S$:

- $e_\mathfrak{p} := p^{\nu_p(N(\mathfrak{p})-1)}$ if $N(\mathfrak{p}) \equiv 1 \pmod p$,
- $e_\mathfrak{p} := p^K$ if $\mathfrak{p}|p$ and the p^K-th roots of unity are in K and the p^{k+1}-th roots of unity are not in K,
- $e_\mathfrak{p} := 2$ if $\mathfrak{p}$ is real.

Let

$$R \to F \to G_S \tag{1}$$

be a minimal representation of G_S by the free pro-p-group F. We construct it by means of a minimal system $\{\overline{\tau}_i | i \in I\}, \tau_i \in G_S$ of generators of $G_K/G_K^{(2)}G_K^p \cong \mathfrak{C}_S/\mathfrak{C}_S^p$. Let F be the free pro-p-group with generator system $\{t_i | i \in I\}$, and let θ be a continuous section of G_K into F with $\theta\tau_i = t_i$.

For any $\mathfrak{p} \in S$ we choose an extension $\mathfrak{P}$ in K_S and we denote the decomposition group of $\mathfrak{P}$ by $D_\mathfrak{p}$. Let $G_\mathfrak{p}$ be the Galois group of the maximal p-extension $\overline{K}_\mathfrak{p}$ of $K_\mathfrak{p}$. The inclusions

$$K_S \to (K_S)_\mathfrak{P} \to \overline{K}_\mathfrak{p}$$

induce a homomorphism $\psi_\mathfrak{p}$ of $G_\mathfrak{p}$ into G_S.

Let

$$R_\mathfrak{p} \longrightarrow F_\mathfrak{p} \xrightarrow{\varphi_\mathfrak{p}} G_\mathfrak{p} \tag{2}$$

be a minimal presentation of $G_{\mathfrak{p}}$ by the free pro-p-group $F_{\mathfrak{p}}$. Then we have a diagram

$$\begin{array}{ccccc} R_{\mathfrak{p}} \to & F_{\mathfrak{p}} \to & G_{\mathfrak{p}} \\ \downarrow & \downarrow & \downarrow \\ R \to & F \to & G_S, \end{array} \tag{3}$$

where the map from $F_{\mathfrak{p}}$ into F is $\chi_{\mathfrak{p}} := \psi_{\mathfrak{p}}\varphi_{\mathfrak{p}}$.

The presentations (2) of $G_{\mathfrak{p}}$ are well known (see e.g. [Ko1970] for references). For our purpose we need in the moment only the fact that $R_{\mathfrak{p}}$ is generated as closed normal subgroup of $F_{\mathfrak{p}}$ by one relation $r_{\mathfrak{p}}$ beside the case that $\mathfrak{p}|p$ and $\delta(K_{\mathfrak{p}}) = 0$ where $F_{\mathfrak{p}}$ is free. $r_{\mathfrak{p}}$ has the form

$$r_{\mathfrak{p}} = t_{\mathfrak{p}}^{e_{\mathfrak{p}}} r'_{\mathfrak{p}} \text{ with } r'_{\mathfrak{p}} \in F_{\mathfrak{p}}^{(2)}.$$

Here $t_{\mathfrak{p}}$ is an element of $F_{\mathfrak{p}}$, which is mapped onto $\zeta_{\mathfrak{p}}$ by the homomorphism

$$F_{\mathfrak{p}} \to G_{\mathfrak{p}}/G_{\mathfrak{p}}^{(2)} \to \hat{K}_{\mathfrak{p}}^{\times},$$

where $\zeta_{\mathfrak{p}}$ denotes a primitive root of unity of order $e_{\mathfrak{p}}$ in $K_{\mathfrak{p}}$. We put $r_{\mathfrak{p}} := 1$ in the case $\mathfrak{p}|p, \delta(K_{\mathfrak{p}}) = 0$.

According to [Ko1970], §11, the relation module R in (3) is generated as normal subgroup of F by the local relations $\chi_{\mathfrak{p}} r_{\mathfrak{p}}, \mathfrak{p} \in S$ if $\tilde{\mathcal{B}}_S = \{0\}$. If the p-th roots of unity are contained in K, one arbitrary $\chi_{\mathfrak{p}_\circ} r_{\mathfrak{p}_\circ}$ of these relations can be omitted. We put $S^* := S$ if $\delta(K) = 0$, $S^* := S - \{\mathfrak{p}_\circ\}$ if $\delta(K) = 1$.

2. In this section we use the methods and some results of [KoKuLa]. If F is a free pro-p-group, then $L(F)$ can be identified with the free graded Lie algebra over the pro-p-group $L_1(F) = F/F^{(2)}$. We study the map

$$\phi_2 : \mathcal{L}_2(\hat{\mathfrak{C}}_S) \longrightarrow L_2(G_S)$$

by means of the two maps

$$\psi_2 : L_2(F) \longrightarrow L_2(G_S)$$

and

$$T_2 : L_2(F) \longrightarrow \mathcal{L}_2(\hat{\mathfrak{C}}_S).$$

T_2 is defined by

$$L_2(F) = \mathcal{L}_2(F/F^{(2)}) \longrightarrow \mathcal{L}_2(G_S/G_S^{(2)})$$

and the map

$$\mathcal{L}_2(G_S/G_S^{(2)}) \longrightarrow \mathcal{L}_2(\hat{\mathfrak{C}}_S),$$

induced by the inverse of the Artin map ϕ_1.

We begin with ψ_2. Let $R^{(2)} := R \cap F^{(2)}$. Then

$$\text{Ker } \psi_2 = R^{(2)} F^{(3)} / F^{(3)}.$$

Proposition 0.2. *Let $\mathcal{B}_S = \{0\}$ and $\mathfrak{p} \in S$ for one place $\mathfrak{p}|p$. Then Ker ψ_2 is generated as pro-p-group by the classes*

$$[\chi_\mathfrak{p} t_\mathfrak{p}^{e_\mathfrak{p}}, x] F^{(3)}, \ \ \mathfrak{p} \in S^*, \ \ x \in F. \tag{4}$$

Proof. Since R is generated as normal subgroup of F by the relations $\chi_\mathfrak{P} r_\mathfrak{p}$, $\mathfrak{p} \in S^*$, it is clear, that the classes (4) are in $R^{(2)} F^{(3)}/F^{(3)}$. It remains to shows that they generate $R^{(2)} F^{(3)}/F^{(3)}$.

Any class of $R^{(2)} F^{(3)}/F^{(3)}$ is of the form

$$\prod_{\mathfrak{p} \in S^*} (\chi_\mathfrak{p} r_\mathfrak{p})^{\alpha_\mathfrak{p}} r F^{(3)}, \alpha_\mathfrak{p} \in \mathbb{Z}_p,$$

with

$$\prod_{\mathfrak{p} \in S^*} (\chi_\mathfrak{p} r_\mathfrak{p})^{\alpha_\mathfrak{p}} \in F^{(2)}, \tag{5}$$

where $rF^{(3)}$ is in the pro-p-group generated by the elements (4).

For the proof of Proposition 0.2 it is sufficient to show $\alpha_\mathfrak{p} = 0$ for $\mathfrak{p} \in S^*$. (5) is equivalent to

$$\prod_{\mathfrak{p} \in S^*} (\chi_\mathfrak{p} t_\mathfrak{p})^{e_\mathfrak{p} \alpha_\mathfrak{p}} \in F^{(2)}. \tag{6}$$

3. Now we have our problem reduced to a question of class field theory. We have to study $G_S/G_S^{(2)} \cong \hat{\mathfrak{C}}_S$ in connection with $F/F^{(2)}$.

First we construct a generator system of $G_S/G_S^{(2)}$ according to [Ko1970], §11.4. We begin by completing our local considerations in 1.

If $N(\mathfrak{p}) \equiv 1 \pmod{p}$ or $\mathfrak{p}$ real, the inertia group of $G_\mathfrak{p}$ is cyclic. We denote the element corresponding to $\zeta_\mathfrak{p}$ in $\hat{\mathfrak{C}}_S$ again by $\zeta_\mathfrak{p}$ and the corresponding element in $G_S/G_S^{(2)}$ by $\tau'_\mathfrak{p}$.

For $\mathfrak{p}|p$ let $\zeta_\mathfrak{p}, \alpha_{1\mathfrak{p}}, \dots, \alpha_{n_\mathfrak{p}\mathfrak{p}}$ with $n_\mathfrak{p} := [K_\mathfrak{p} : \mathbb{Q}_\mathfrak{p}]$ be a basis of the principal unit group of $K_\mathfrak{p}$. We denote the corresponding elements in $\hat{\mathfrak{C}}_K$ again by $\zeta_\mathfrak{p}, \alpha_{1\mathfrak{p}}, \dots, \alpha_{n_\mathfrak{p}\mathfrak{p}}$ and the corresponding elements in $G_S/G_S^{(2)}$ by $\tau'_\mathfrak{p}, \tau'_{1\mathfrak{p}}, \dots, \tau'_{n_\mathfrak{p}\mathfrak{p}}$.

Furthermore, let $Cl(K)(p)$ be the p-component of the ideal class group of K and let $\mathfrak{a}_1, \dots, \mathfrak{a}_h$ be ideles in I_K such that the corresponding ideals $\mathfrak{a}'_1, \dots, \mathfrak{a}'_h$

form a basis of $Cl(K)(p)$. Let $\omega_1, \dots, \omega_h$ be the corresponding elements in $G_S/G_S^{(2)}$. Then by class field theory

$$\{\omega_1, \dots, \omega_h\} \cup \{\tau'_{\mathfrak{p}} | \mathfrak{p} \in S\} \cup \{\tau'_{1\mathfrak{p}}, \dots, \tau'_{n_{\mathfrak{p}}\mathfrak{p}} / \mathfrak{p} \in S, \mathfrak{p}|p\}$$

is a generator system of $G_S/G_S^{(2)}$.

We present $G_S/G_S^{(2)}$ as the quotient of the free abelian pro-p-group A with generator system

$$\{w_1, \dots, w_h\} \cup \{t'_{\mathfrak{p}} | \mathfrak{p} \in S\} \cup \{t'_{1\mathfrak{p}}, \dots, t'_{n_{\mathfrak{p}}\mathfrak{p}} | \mathfrak{p} \in S, \mathfrak{p}|p\}.$$

We put $t'_{\mathfrak{p}} = 1$ if $\mathfrak{p}|p, \delta(K_{\mathfrak{p}}) = 1$. With the map $\alpha : A \to G_S/G_S^{(2)}$ given by

$$\begin{aligned} \alpha(w_i) &= \omega_i, i = 1, \dots, h, \alpha(t'_{\mathfrak{p}}) = \tau'_{\mathfrak{p}}, \mathfrak{p} \in S, \\ \alpha(t'_{i\mathfrak{p}}) &= \tau'_{i\mathfrak{p}}, i = 1, \dots, n_{\mathfrak{p}}, \mathfrak{p}|p. \end{aligned}$$

Then the kernel of α is generated by the local relations

$$t'^{e_{\mathfrak{p}}}_{\mathfrak{p}}, \mathfrak{p} \in S \tag{7}$$

and the relations which correspond to the finiteness of $Cl(K)(p)$ and the group E of units of K:

Let $\varepsilon \in E$. Then we have the following equation in $\hat{I}_K$:

$$(\varepsilon) = \prod_{\mathfrak{p} \in S} \zeta_{\mathfrak{p}}^{a_{\mathfrak{p}\varepsilon}} \prod_{\substack{\mathfrak{p}|p \\ \mathfrak{p} \in S}} \prod_{i=1}^{n_{\mathfrak{p}}} \alpha_{i\mathfrak{p}}^{b_{i\mathfrak{p}\varepsilon}}, \;\; a_{\mathfrak{p}\varepsilon} \in \mathbb{Z}, \;\; b_{i\mathfrak{p}} \in \mathbb{Z}_p.$$

Let q_j be the order of $\mathfrak{a}'_j$ and $\beta_j \in K^\times$ such that $\mathfrak{a}_j'^{q_j} = (\beta_j), j = 1, \dots, h$. Then $\mathfrak{a}_j^{q_j}(\beta_j^{-1})$ is a unit idele whose image in $\hat{I}_K$ has the form

$$\prod_{\mathfrak{p} \in S} \zeta_{\mathfrak{p}}^{c_{\mathfrak{p}j}} \prod_{\substack{\mathfrak{p}|p \\ \mathfrak{p} \in S}} \prod_{i=1}^{n_{\mathfrak{p}}} \alpha_{i\mathfrak{p}}^{d_{i\mathfrak{p}j}}, \;\; c_{\mathfrak{p}j} \in \mathbb{Z}, \;\; d_{i\mathfrak{p}j} \in \mathbb{Z}_p.$$

$j = 1, \dots, h$. The correspondent relations in A are

$$\prod_{\mathfrak{p} \in S} t'^{a_{\mathfrak{p}\varepsilon}}_{\mathfrak{p}} \prod_{\substack{\mathfrak{p}|p \\ \mathfrak{p} \in S}} \prod_{i=1}^{n_{\mathfrak{p}}} t'^{b i\mathfrak{p}\varepsilon}_{i\mathfrak{p}} \tag{8}$$

and

$$w_j^{-q_j} \prod_{\mathfrak{p} \in S} t'^{c_{\mathfrak{p}j}} \prod_{\substack{\mathfrak{p}|p \\ \mathfrak{p} \in S}} \prod_{i=1}^{n_{\mathfrak{p}}} t'^{b_{i\mathfrak{p}\varepsilon}}. \tag{9}$$

The relations (7) - (9) generate Ker α by construction.

Now we have to relate A and α to $F/F^{(2)}$.

$F/F^{(2)}$ is to be generated by a minimal system of generators. We can do this by choosing among the generators of A a minimal subsystem $\{t_i | i \in I\}$. This defines a homomorphism β of A onto $F/F^{(2)}$ with kernel generated by the relations (8) and (9). This corresponds to the procedure in [Ko1970], §11.4.

4. Now we come back to the proof of Proposition 0.2. The condition (6) is equivalent to

$$\prod_{\mathfrak{p}\in S^*} {t'}_{\mathfrak{p}}^{e_\mathfrak{p}\alpha_\mathfrak{p}} \in \ \mathrm{Ker}\beta. \tag{10}$$

Since the generators $\omega_1, \dots, \omega_h$ do not appear in (10), the relation has to be of the form

$$\prod_{\mathfrak{p}\in S} {t'}_{\mathfrak{p}}^{a_\mathfrak{p}\varepsilon} \prod_{\substack{\mathfrak{p}|p\\ \mathfrak{p}\in S}} \prod_{i=1}^{n_\mathfrak{p}} {t'}_{i\mathfrak{p}}^{b_{i\mathfrak{p}\varepsilon}} \tag{11}$$

for an $\varepsilon \in E$. By assumption $\mathfrak{p}_\circ \in S$, $\mathfrak{p}_\circ | p$, hence $b_{i\mathfrak{p}_\circ\varepsilon} = 0$ for $i = 1, \dots, n_{\mathfrak{p}_\circ}$. This implies that ε is a root of unity. Since only powers of p contribute to (11), we can assume that ε is a root of unity of p-th power order. In the case $\delta(K) = 0$ this implies $a_{\mathfrak{p}\varepsilon} = 0$ and therefore $\alpha_\mathfrak{p} = 0$. In the case $\delta(K) = 1$ we let the product in (10) run through $S^* = S - \{\mathfrak{p}_\circ\}$. This means $a_{\mathfrak{p}_\circ\varepsilon} = 0$ and $\varepsilon = 1$, hence $\alpha_\mathfrak{p} = 0$. This proves Proposition 0.2. □

5. Now we go over to the consideration of Ker T_2.

Proposition 0.3. *Ker* $T_2 =$ *Ker* ψ_2.

Proof. Since $\hat{\mathfrak{C}}_S \cong G_S/G_S^{(2)}$, the assertion is equivalent to Ker $T_2' =$ Ker ψ_2, where T_2' denotes the natural map

$$T_2' : L_2(F) \longrightarrow \mathcal{L}_2(G_S/G_S^{(2)}).$$

It follows immediately from Proposition 1 that Ker ψ_2 is contained in Ker T_2'. On the other hand ψ_2 factors through T_2' and therefore Ker $T_2' \subseteq$ Ker ψ_2. □

6. Now it is easy to prove Theorem 1. We have a triangle of surjective maps

$$\begin{array}{ccc} & L_2(F) & \\ \swarrow T_2 & & \searrow \psi_2 \\ \mathcal{L}_2(\hat{\mathfrak{C}}_S) & \xrightarrow{\phi_2} & L_2(G_S). \end{array}$$

Since Ker $T_2 =$ Ker ψ_2 the map ϕ_2 is a isomorphism. □

7. Now we study the arithmetic properties of ϕ_2. Let $\mathfrak{p}$ be a place of K and $\mathfrak{P}$ an extension of $\mathfrak{p}$ to the maximal p-extension. We denote by $D_{\mathfrak{P}}$ the decomposition group of $\mathfrak{P}$ in G_S.

Proposition 0.4. *If the natural map $\hat{K}_{\mathfrak{p}}^{\times} \to \hat{\mathfrak{C}}_s$ is an injection, then*

$$D_{\mathfrak{P}} \cap G_S^{(2)} = D_{\mathfrak{P}}^{(2)}. \tag{12}$$

Proof. Let $G_{\mathfrak{p}}$ be the Galois group of the maximal p-extension of $K_{\mathfrak{p}}$. Then by our assumption the map

$$G_{\mathfrak{p}}/G_{\mathfrak{p}}^{(2)} \longrightarrow G_S/G_S^{(2)}$$

is injective. Since the image of the restriction map $\psi_{\mathfrak{p}} = G_{\mathfrak{p}} \to G_S$ is $D_{\mathfrak{P}}$, (12) follows immediately. □

Remark 0.5. The assumption that $\hat{K}_{\mathfrak{p}}^{\times} \to \hat{\mathfrak{C}}_S$ is injective is fulfilled if $\overline{K} = K_S$ is the maximal p-extension.

The decomposition group of $\mathfrak{P}$ in $G_S^{(n)}/G_S^{(n+1)}$ is $(D_{\mathfrak{P}} \cap G_S^{(n)})G_S^{(n+1)}/G_S^{(n+1)}$.

Proposition 0.6. *The decomposition group of $\mathfrak{P}$ in $G_S^{(n)}/G_S^{(n+1)}$ is independent of the choice of the extension $\mathfrak{P}$ of $\mathfrak{p}$.*

Proof. Let $\mathfrak{P}'$ be any extension of $\mathfrak{p}$ to K_S. Then there is an $\xi \in G_S$ such that $\mathfrak{P}' = \xi\mathfrak{P}$ and $D_{\mathfrak{P}'} = \xi D_{\mathfrak{P}} \xi^{-1}$.

If $\eta \in D_{\mathfrak{P}} \cap G_S^{(n)}$, then $\xi\eta\xi^{-1} \in D'_{\mathfrak{P}} \cap G_S^{(n)}$ and

$$\xi\eta\xi^{-1} \equiv \eta (\mathrm{mod}\ \ G_S^{(n+1)}).$$

□

We denote the decomposition group of $\mathfrak{P}$ in $L_n(G_S)$ by $L_{n\mathfrak{p}}(G_S)$.

To the map $K_{\mathfrak{p}}^{\times} \to \hat{\mathfrak{C}}_S$ there corresponds a map $\Lambda_{\mathfrak{p}} : \mathcal{L}_2(K_{\mathfrak{p}}^{\times}) \to \mathcal{L}_2(\hat{\mathfrak{C}}_S)$.

Theorem 0.7. *If the natural map $\hat{K}_{\mathfrak{p}}^{\times} \to \hat{\mathfrak{C}}_S$ is an injection, then*

$$\phi_2\Lambda_{\mathfrak{p}}\mathcal{L}_2(\hat{K}_{\mathfrak{p}}^{\times}) = L_{2\mathfrak{p}}(G_S).$$

Proof. It is clear that the map $\phi_2\Lambda_{\mathfrak{p}}$ maps $\mathcal{L}_2(K_{\mathfrak{p}}^{\times})$ into $L_{2\mathfrak{p}}(G_S)$. By Proposition 3 we have

$$L_{2\mathfrak{p}}(G_S) = D_{\mathfrak{P}}^{(2)} G_S^{(3)}/G_S^{(3)}.$$

On the other hand we see from the commutative diagram

$$\begin{array}{ccc} \mathcal{L}_2(K_{\mathfrak{p}}^{\times}) & \xrightarrow{\Lambda_{\mathfrak{p}}} & \mathcal{L}_2(\hat{\mathfrak{C}}_S) \\ \downarrow & & \downarrow \\ L_2(G_{\mathfrak{p}}) & \longrightarrow & L_2(G_S) \end{array}$$

that

$$\phi_2\Lambda_{\mathfrak{p}}\mathcal{L}_2(K_{\mathfrak{p}}^{\times}) = D_{\mathfrak{P}}^{(2)} G_S^{(3)}/G_S^{(3)}.$$

□

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